Physics 15b, Lab 3: The Capacitor… and a glimpse of Diodes

REV0[1]; March 14, 2007

Due Friday, March 23, 2007.

NOTE that this is the first of the labs that you are invited to do in your own room, if you like, although we will hold *help labs* on Wednesdays and Thursdays:

- Wednesday: 6-9 p.m. (March 21)
- Thursday: 3-9 p.m. (March. 22)

There will be *no* lab sections, apart from these three help sessions.

You can do this experiment alone, but we encourage you to do it as a partnership, taking data collaboratively.

Contents

1 Purpose

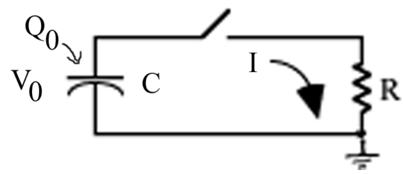
To study the charging and discharging of a capacitor in an RC circuit, with emphasis on the time dependence of the voltage and current. You should also think about how the capacitor stores energy in the electric field, and how this energy can be added and subtracted from the capacitor.

You will also get a look at the way a diode can *rectify* an AC waveform, and with the help of a capacitor, can begin to produce a DC voltage from an AC sinusoid. This is a process you will look at more closely in Lab 4.

[1] Revisions: add table of contents (3/06); ground added to figures, because questions refer to "voltages" not $V_{\rm CAP}$, etc.; cap % typo fixed, p. 3; most of this lab was called "Lab 4" till this term. This is old Lab 4 minus ripple, but adding in Lab 3's introduction to diodes (old 3.1) (10/04).

2 Background

Consider the voltage and the current as a function of time for a capacitor charged to voltage V_0, discharging through a resistor:



This problem is discussed in section 4.11 of Purcell. We want to find the voltage as a function of time, and we know three relationships:

$$Q = CV \qquad I = \frac{V}{R} \qquad I = \frac{dQ}{dt}$$

We can use these to write a differential equation for V, and integrate to find V as a function of time. Showing all the steps:

$$-\frac{dQ}{dt} = -C\frac{dV}{dt} = \frac{V}{R} \tag{1}$$

$$\int_{V_0}^{V} \frac{dV}{V} = -\int_0^t \frac{dt'}{RC} \tag{2}$$

$$ln\frac{V(t)}{V_0} = -\frac{t}{RC} \tag{3}$$

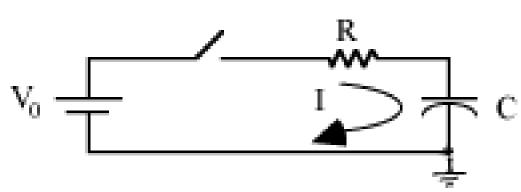
$$V(t) = V_0 e^{-\frac{t}{RC}} \tag{4}$$

In words: the voltage starts at V0, and decays toward zero.

Since $I = \frac{V}{R}$, we also have: $I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}$

Notice that the variation of V and I with time depends on the product RC in the exponential. This quantity, which is called the time constant (τ), is equal to the time for decay to 1/e times the initial value.

A related problem is the charging of a capacitor through a resistor:



Now we have the three relations:

$$Q = CV \qquad I = \frac{V_0 - V}{R} \qquad I = \frac{dQ}{dt} \tag{5}$$

Again we can write a differential equation for V and solve it. Without showing all the steps, which are similar to the example above:

$$C\frac{dV}{dt} = \frac{V_0 - V}{R} \tag{6}$$

$$\int_0^V \frac{dV'}{V_0 - V'} = -\int_0^t \frac{dt'}{RC} \tag{7}$$

$$V = V_0(1 - e^{-\frac{t}{RC}}) \tag{8}$$

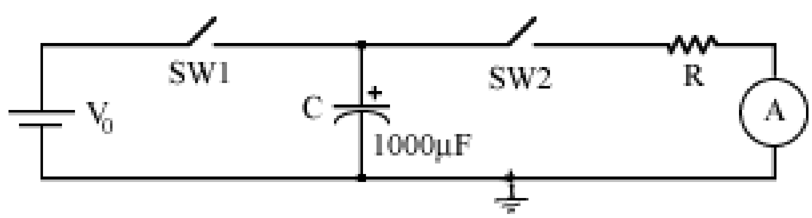
Since $I = \frac{V_0 - V}{R}$, we have:$I(t) = \frac{V_0}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$
In words: when you close the switch, the current starts at V_0/R and decays toward zero.

In this experiment we hope you will be able to demonstrate the rules stated above.

3 Procedure

3.1 Discharge of a capacitor

You can use the circuit below, momentarily closing and then opening SW1 (the "switches," "SW1" and "SW2" can be simply wires touched to the breadboard, then released) so as to charge the capacitor, with "SW2" open. Then close SW2 to begin the discharge, measuring t from the time you close SW2. More elegantly, you can close both SWl and SW2, observing a steady current $I_0 = V_0/R$ through your meter, and then measure the decrease of the current as the capacitance discharges after you open SW1.



Question 1: Let C be your unused 1000 μF capacitor. If R were 1 kΩ, what would the time constant be? Pick a resistor out of your unused ones that will give a convenient time constant (of order one minute) for following the current change as the capacitor discharges. Calculate the maximum current for voltages you can easily get with your dry cells, and select a value of V_0 appropriate for use with your 50 μA meter scale.

Question 2: What is the meter resistance on the 50 μA scale? On this scale (where the small fuse resistance is negligible relative to the meter movement's resistance), you can count on the voltage across the meter to be 250 mV full-scale, 125 mV half-scale, etc. What correction do you need to make?

Record I as a function of time, and plot it. If you had semi-log paper, you could use it to verify exponential decay simply by looking for a straight line. If you use the linear paper of your notebook, instead (much more likely; few of you carry around semi-log paper!) you can achieve the same result, by plotting the $\log I$ vs t. (Do you want to use the natural log (ln), or log base 10? Does it matter what units you use – i.e. ln(μA) vs ln(A)?)

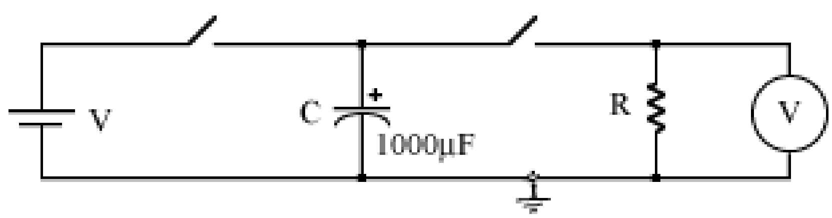
Estimate the time constant You can estimate the time-constant, tau ($\tau = RC$), from the graph you have just drawn. Since R is known (to $\pm 5\%$) you can calculate a value for C. Recognize that the error in C comes from two sources:

- error in R: 5%;
- error in your measurement of τ (caused by uncertainties in your measurements of time and voltage).

Combine these errors appropriately to calculate a resulting percentage uncertainty for C.

Compare your calculated value of C against the capacitor's *nominal* value. The C value may look wrong, at first. But electrolytic capacitors like the one you are using show wide and *asymmetric* variation from the nominal value: usually they are used in uncritical applications such as filters (The asymmetry may puzzle you: it reflects what customers want in a filter cap: they are rate -20% to +100%. This scheme is not as crazy as it may seem, at first glance: this kind of capacitor is used in uncritical applications such as filters; and the asymmetry reflects what customers want in a filter cap: they'll complain if C is undersized, because the cap's smoothing effect will be less than expected; they won't complain if the smoothing is a little more than expected. Cap manufacturers know this, and spec their parts accordingly.)

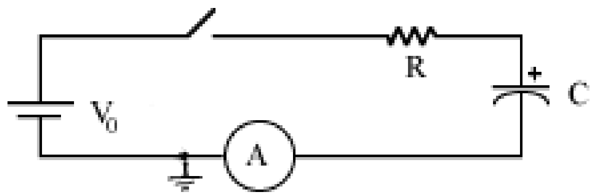
Now record V as a function of time, on the 5 volt scale of your meter:



and use this curve to calculate C. On the 5 volt scale the voltmeter has a resistance of $100,000\ \Omega$. Correct for its effect and compare C with that determined previously.

3.2 Charging of a capacitor

Use the same R and C as above, and check that I as a function of time is consistent with your expectations. Make sure to start with the capacitor fully discharged! You'll notice that inserting the ammeter makes $V\_mathrm0$ not quite equal to the voltage applied to the RC: the ammeter soaks up 1/4V, full-scale.

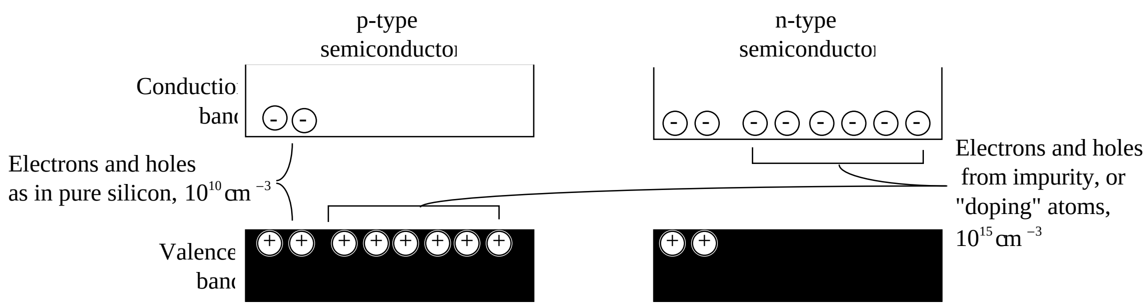


First Look at Diodes

4 Diodes: Background

4.1 Theory

Last week you studied the I-V curve of a semiconductor light-emitting diode, and saw that it was a good conductor in only one direction. Not all semiconductor diodes emit light, but they do exhibit unidirectional current flow. To understand this, first read *Purcell, Sec. 4.6.*. Figure 4.11 (repeated below) shows diagrammatically that in n-type silicon the dominant mobile charge carriers - called majority carriers - are electrons in the conduction band, while in p-type silicon the majority carriers are "holes" in the valence band. Note that the carriers generated by thermal excitation (called electrons and holes in pure silicon in the figure) are enormously fewer, and they can be ignored.



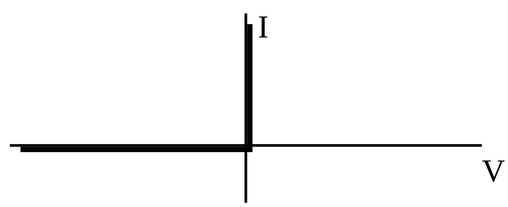
It is possible to dope a single crystal of silicon so that it contains a planar boundary with p-type silicon on one side and n-type on the other. At this "p-n junction", it is as if the n-type and p-type regions in the figure above were moved together sideways so as to touch. Notice that an electric field to the right would cause motion of both signs of dominant carriers toward the junction, where they could recombine. Note particularly that this motion corresponds to conventional current flow to the right in both regions. Imagine wires bonded to the silicon at each side of the figure, which can inject electrons at the right into the n-type material, and holes at the left into the p-type material, and you can see that this "easy" current flow can continue steadily in a circuit. In contrast, consider an electric field to the left. Now both signs of the dominant charge carriers are pulled away from the junction, no recombination takes place, and no current can flow.

Referring to the LED experiment last week, this discussion qualitatively describes what you observed: easy current flow in the "forward" direction and little or no current flow in the "reverse" direction. In addition, you saw light associated with the current flow in the forward direction. This comes from photons given off during recombination at the junction. The semiconductor materials used to create the p-n junction determine whether light is emitted, and if so of what color. For silicon diodes, no light is emitted.

The discussion above ignores the junction recombination of dominant carriers - even with no current flow - which produces a double layer of charges, as discussed in Purcell's problem 4.13. However, although the quantitative behavior of the junction is more complex, the qualitative features described above are correct.

4.2 Rectifiers

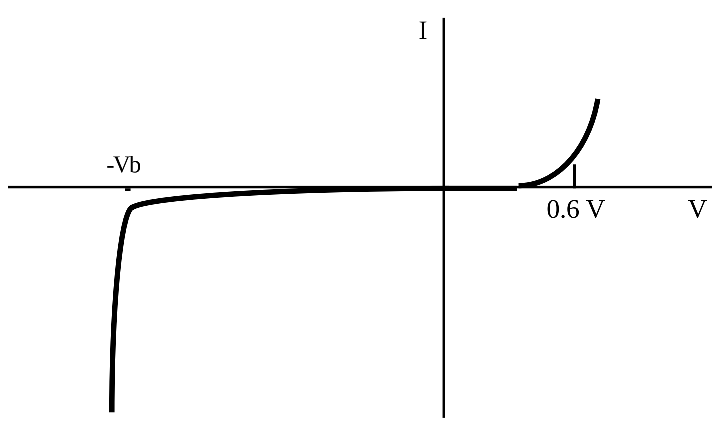
One important use of diodes is in circuits called rectifiers, used to convert alternating voltages to direct voltages. To explain the use of a diode in a rectifier, one can idealize its I-V curve as shown below:



The device can be thought of as a perfect unidirectional conductor: a one-way valve.

4.3 *Optional Theory*

Real diodes are a little more complicated, and the graph of I versus V actually looks more like this:



There is a *breakdown* voltage ($-V_b$) beyond which the diode will conduct even in the backward direction. (This $-V_b$ is around -100V for an ordinary silicon diode, so you'll not see it in our labs. The breakdown voltage is much lower, however, for last week's LED: the spec sheet says it can be as low as -4V.) For positive I and V the graph does not look quite like an "L;" more exactly, it is an exponential of the form:

$$I(V) = I_o(e^{qV/kT} - 1)$$

or, at room temperature: $I(V) approx. = I_o(e^{V/25mV})$

In the first equation, q is the (positive) electron charge, k is Boltzmann's constant, and T is the absolute temperature. At room temperature $\frac{kT}{q} \cong 25\ mV$, as we have suggested already. I_0, the reverse leakage current, depends on the material from which the diode is made. Typically it is a few hundredths of a μA, so the reverse current is almost never of any consequence (until of course one reaches the breakdown voltage). In the forward direction an ordinary silicon diode begins to conduct milliamps at about 0.6 volts (at room temperature). To put it another way, if current is flowing, the voltage across the diode is around 0.6 volts, not zero.

Although a forward-biased diode does not have a resistance in the usual sense of the word (I is not proportional to V), it is occasionally useful to speak of a **dynamic resistance** (r), defined as the ratio of a small change in V to the resulting small change in **I** (this you may recall from a note in Experiment 1):

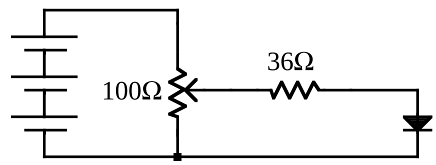
$$r = \frac{dV}{dI} \cong \frac{kT}{qI}$$

note that this is simply the inverse of the slope of the I-V curve at any point.

*** End of optional theory ***

5 Diode Experiment: Silicon diode $I - V$ curve

You have two high-voltage silicon diodes. They are small gray-and-black ceramic blobs in the diode section of your parts box. The silicon is inside. Using the simple circuit below (or something of your own design, if you prefer), measure the I-V curve of one of them.



You can use the circuit above to collect I,V pairs for the diode. Your meter makes it hard to read currents that span a wide range, as they should for this diode exercise. The meter provides only two ranges–50 microamps and 250 milliamps full-scale: ranges that differ by a factor of five thousand! But a simple exploitation of Ohm's Law can let you bridge this huge gap, as suggested in §5.1, below.

Compare the result with that for your LED. As with the LED, try both current directions, and note any points of interest. Does the black stripe correspond to the arrow (called the anode) or the line (the cathode) on the schematic of a diode? The arrow and stripe form the standard way of indicating the diode polarity: conventional current (*not* electron flow!) travels in the direction of the arrow's pointing.

5.1 A Trick to fill in missing Current Ranges on your meter

If you use the meter as *voltmeter* (on its most sensitive scale: full-scale: 250mV), and read the voltage across the resistor that is in series with the diode, you can use Ohm's Law to infer the diode current.If you would like to read smaller currents, you can use a larger resistor: in effect, you are filling in some current scales that the meter's designers omitted.

Here are some sample full-scale current ranges you could contrive, given some R's in your kit:

Series R (ohms)	I full-scale (mA)
1	250
2.7	93
18	14
36	6.9
360	0.7

These numbers are pretty nasty to work with, because the R values aren't nice and round. If you come to *help* lab, however, we can provide nicer values: round numbers like 10Ω and 100Ω, for example. For these values, the current is easy to read from the meter, because the full-scale values are so tidy: 25mA and 2.5mA, for 10Ω and 100Ω, respectively.

Don't let this suggestion drive you to pick up a zillion I-V points; but you might enjoy this little Ohm's Law workout!