

## ■ *Mathematica* for Physics - Chapter 2, Section 1, Problem 3: Projectile with Air Resistance ( $F_{\text{drag}} = -bv$ )

```
(Local) In[41]:=
Clear["Global`*"];
```

### ■ Solve the equations of motion for the projectile.

```
(Local) In[42]:=
eqm = {m v'[t] == -m g - b v[t], y'[t] == v[t]};
```

```
(Local) In[43]:=
init = {y[0] == y0, y'[0] == v0};
```

```
(Local) In[44]:=
dsol = DSolve[Join[eqm, init], {v, y}, t] // Flatten // Simplify;
{v[t], y[t]} /. dsol
```

```
(Local) Out[45]=
{ - (m g + m b v0) / b, 1 / b^2
  (m (-g m^2 + m g m^2 - b m g t - b m v0 + b m v0 + b^2 m v0 + b^2 m y0)) }
```

### ■ Find the terminal velocity for the projectile.

```
(Local) In[46]:=
vterm = v[t] /. dsol /. {t -> large m / b} // ExpandAll
```

```
(Local) Out[46]=
- g m / b + large g m / b + large v0
```

```
(Local) In[47]:=
      vterm = Limit [vterm, large  $\infty$  Infinity]
```

```
(Local) Out[47]=
      -  $\frac{g m}{b}$ 
```

■ Take the limit as  $b \rightarrow 0$  (no air resistance).

```
(Local) In[48]:=
      {ynar[t], vnar[t]} = Limit[{y[t], v[t]} /. dsol, b  $\infty$  0]
```

```
(Local) Out[48]=
      { -  $\frac{g t^2}{2}$  + t v0 + y0, -g t + v0 }
```

■ Plot the particle's position at equal intervals of time for  $b \neq 0$  and  $b = 0$ .

```
(Local) In[49]:=
      val1 = {g  $\infty$  9.8, m -> 1, y0  $\infty$  0, v0  $\infty$  0};
```

```
(Local) In[50]:=
      coord1[b_] = {t, y[t]} /. dsol /. val1 // Simplify
```

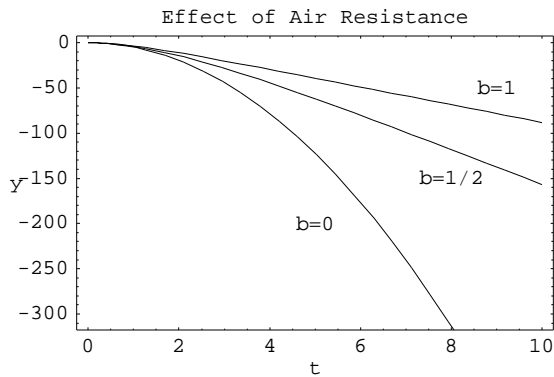
```
(Local) Out[50]=
      {t,  $\frac{g t^2 - 2 b t + 2 y_0}{2 b}$ }
```

```
(Local) In[51]:=
      coord1[0] = {t, ynar[t]} /. val1
```

```
(Local) Out[51]=
      {t, -4.9 t^2}
```

```
(Local) In[52]:=
      Clear[plot1];
      plot1[b_] := ParametricPlot[Evaluate[coord1[b], {t, 0, 10}]];
      plotarray1 = Table[plot1[b], {b, 0, 1, 1/2}];
```

```
(Local) In[54]:=
Show[plotarray1,
Graphics[{Text["b=0", {5, -200}], Text["b=1/2", {8, -150}], Text["b=1", {9, -50}]}],
Frame &#x27; True, FrameLabel &#x27; {"t", "y", "Effect of Air Resistance", ""}];
```



■ Plot the speed for finite and zero air resistance. Show the terminal velocity.

```
(Local) In[55]:=
coord2[b_] = {t, v[t]} /. dsol /. val1 // ExpandAll
coord2[0] = {t, vvar[t]} /. val1;
```

```
(Local) Out[55]=
{t, -  $\frac{g}{b} + \frac{g}{b} e^{-bt}$ }
```

```
(Local) In[57]:=
coord3[b_] = {t, vterm} /. dsol /. val1 // ExpandAll
coord3[0] = {t, 0};
```

```
(Local) Out[57]=
{t, -  $\frac{g}{b}$ }
```

```
(Local) In[59]:=
Clear[plot2];
plot2[b_] := ParametricPlot[Evaluate[{coord2[b], coord3[b]}], {t, 0, 10}]
plotarray2 = Table[plot2[b], {b, 0, 1, 1/2}];
```

```
(Local) In[62]:= Show[plotarray2,  
Graphics[{Text["b=0", {3.5, -25}], Text["b=1/2", {7, -17}], Text["b=1", {8, -7}]}],  
PlotRange  $\{0, -30\}$ , Frame  $\{True, True\}$ , RotateLabel  $\{False, False\}$ ,  
FrameLabel  $\{t, v, "Effect of Air Resistance", ""\}$ ];
```

