uniform univariate b-splines

- given a sequence of $n+3$ control points p_i from $-1..n+1$ (scalars or points)
- b-spline defines $c(t)$ for t from $0..n$
- between each two integers the segment is a cubic over t.
- segments meet up as C^2 functions of t
- change of control point affects local region
- the control points are only approximated

standard interpretation

- can be evaluated using de-cas-like algorithm
- function is a linear combination of b-spline basis functions
- we will not pursue these interpretations here

uniform subdivision

- we can topologically subdivide the control polygon
    - we will associate a new vertex for each old interior vertex
    - we will associate a new vertex for each old edge

uniform subdiv rule

- lets look at the nbrhood of one old vertex v^i.
- lets call its two vertex neighbors e^i_j
- we will set the new "edge" vertex as

$$e^{i+1}_j = 1/2e^i_j + 1/2v^i$$

- we will set the new "vertex" vertex as

$$v^{i+1} = \sum_j 1/4e^{i+1}_j + 1/2v^i$$

result

- this will give us $d(s)$ for s from $0..2n$
- thm: if we use the above subdiv rules and interpret these as b-spline control points, we will get $c(t) = d(2t)$
- i will not go into the proof of this

repeat

- i now have a "denser" control polygon for the original curve
- let me repeat this recursively a few times
- now i have a very dense control polygon for the same curve
- thm: in the limit, the control polygon will converge to the actual curve
- let me just draw this dense polygon as an approximation for the curve

bivariate uniform b-spline

- we can also consider bivariate functions $c(s,t)$.

- if the functions are tri-valued, we can think of this as a surface in R^3. (called a parametric surface)
- let our input be a grid
- we can extend the univariate uniform b-spline ideas to bivariate to get basis functions.
- bunch of bi-cubic patches
- patches meet up as C^2 functions of s, t
    - C^2 is useful for smooth reflections
- we will not pursue this interpretation here.

bivariate uniform b-spline subdivision

- instead, lets apply one step of horizontal subdivision and one step of vertical subdivision.
- in the end we can rearrange the math and describe the result as follows
- lets look at the nbrhood of one old vertex v^i.
- lets call its four vertex neighbors e^i_j
- we will set each of 4 new “face” vertices f^{i+1} as the average of the 4 vertices around the face
- we will set the new “edge” vertex as

$$e^{i+1}_j = 1/4\left(v^i + e^i_j + f^{i+1}_j + f^{i+1}_{j-1}\right)$$

- we will set the new “vertex” vertex as

$$v^{i+1} = 1/2 v^i + \sum_j 1/16 e^i_j + \sum_j 1/16 f^{i+1}_j$$

- thm: this subdivision process will converge to the bivariate b-spline surface
- so do it a few times and output the mesh

generalize

- what about if i start with an input mesh that is not a grid
- lets generalize the rules as follows:
- lets look at the nbrhood of one old vertex v^i.
- lets call its n vertex neighbors e^i_j
- we will set each of n new “face” verticies f^{i+1} as the average of the m vertices around the face
- we will set the new “edge” vertex as

$$e^{i+1}_j = 1/4\left(v^i + e^i_j + f^{i+1}_j + f^{i+1}_{j-1}\right)$$

- we will set the new “vertex” vertex as

$$v^{i+1} = \frac{n-2}{n} v^i + \frac{1}{n^2}\sum_j e^i_j + \frac{1}{n^2}\sum_j f^{i+1}_j$$

- if the input is a grid, i will get the b-spline

properties

- note: after one subdivision, all faces will be quads
- we will call any vertex with valence $\neq 4$ “extraordinary”

- if all faces are quads, the number of extraordinary vertices stays constant under subdivision
- so away from the e.v.s, in the limit we are just building b-splines
  - so we are getting nice C^2 surface
- thm: even at the e.v.s this process converges
- thm: even at the e.v.s the resulting surface is C^1 smooth
- we call this a subdivision surface

more context

- we just looked at "catmull-clark" subdivision.
- there are other subdivision rules that also work.
- there are also rules that have been designed for triangle meshes.
  - e.v. is non valence 6
  - most used triangle method is called "loop" subdivision.